



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE SENIOR
SERTIFIKAAT***

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

**These marking guidelines consist of 29 pages.
*Hierdie nasienriglyne bestaan uit 28 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

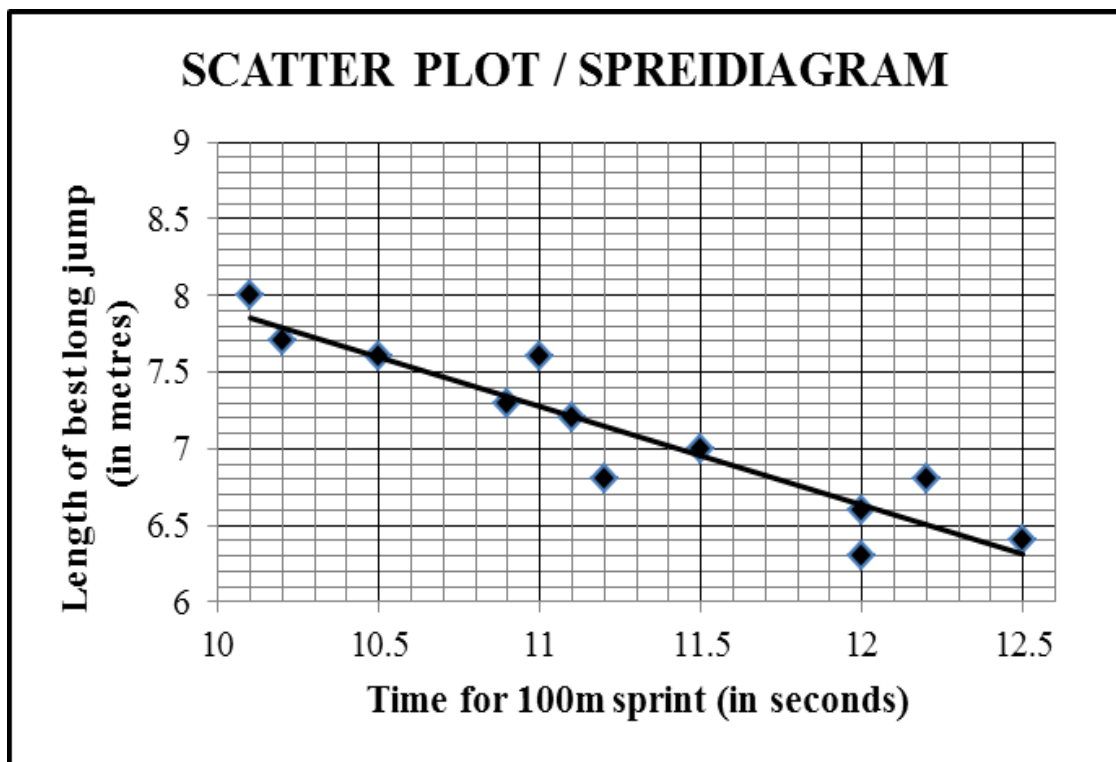
NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason.)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede.)
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is.)
S/R	Award a mark if the statement AND reason are both correct.
	Ken 'n punt toe as beide die bewering EN rede korrek is.

QUESTION/VRAAG 1

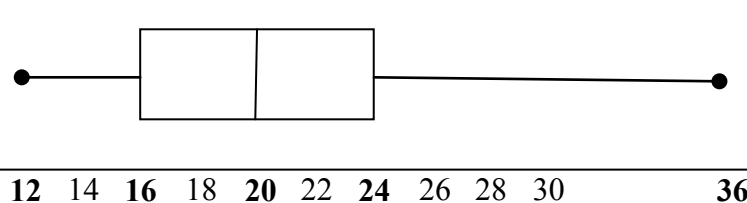
Time for 100 m sprint (in seconds) <i>Tyd vir 100 m-naelloop (in sekondes)</i>	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) <i>Afstand van beste sprong in verspring (in meter)</i>	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4



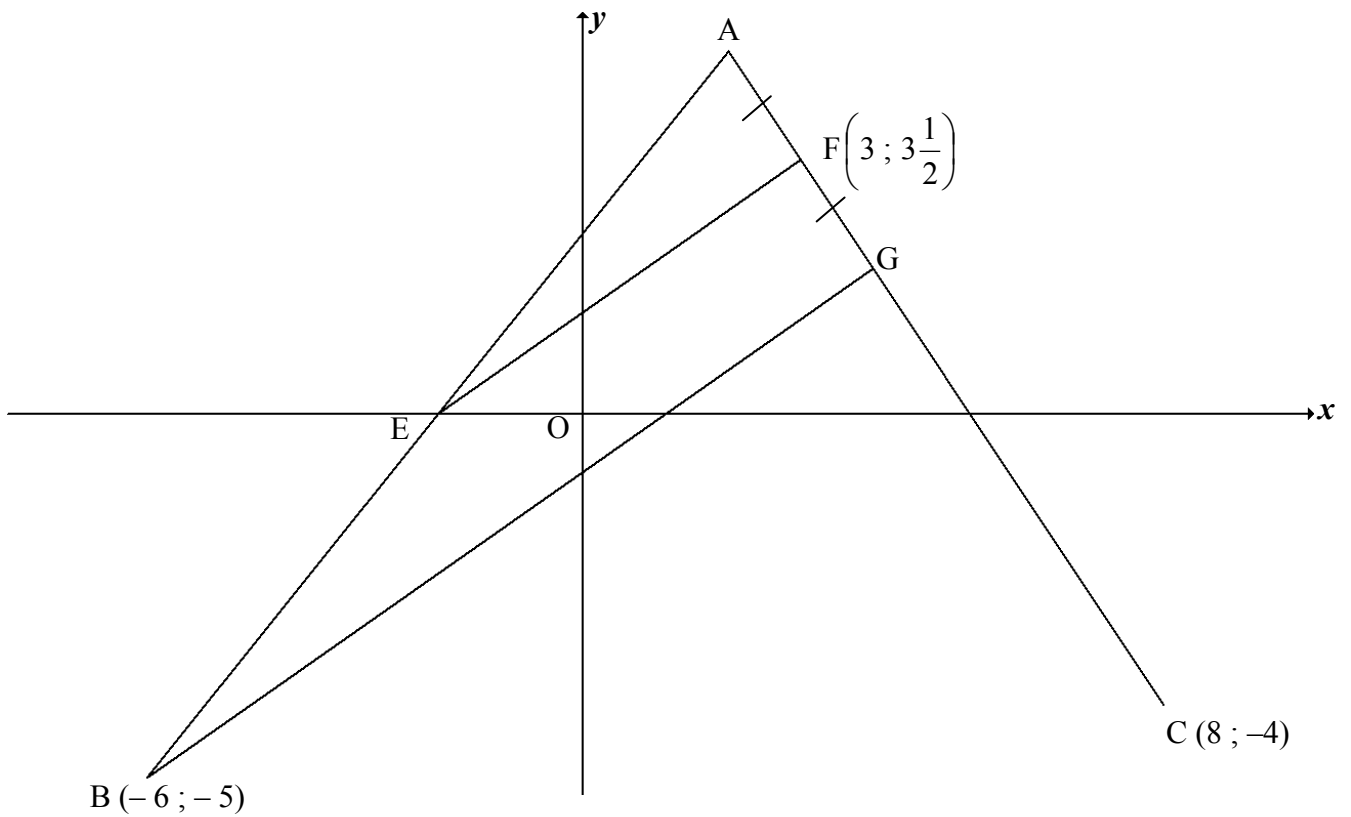
1.1	$a = 14,343\dots = 14,34$ $b = -0,642\dots = -0,64$	✓✓ value of a ✓ value of b (3)
1.2	$y = 14,34 - 0,64(11,7)$ $= 6,85$ OR/OF $y = 6,83$ (calculator / <i>sakrekenaar</i>)	✓ substitution correctly ✓ answer (2) ✓✓ answer (2)
1.3	The gradient increases / <i>Die gradient neem toe</i> The point (12,3 ; 7,6) lies some distance above the current data. / <i>Die punt (12,3 ; 7,6) lê bokant die huidige data.</i>	✓ increases/ <i>neem toe</i> ✓ reasoning in words/ <i>redenasie in woorde</i> (2) [7]

QUESTION/VRAAG 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20,52 \text{ seconds / sekonde}$	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ $IQR/IKO = Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20,52 + 5,94 = 26,46$ $\therefore > 26,46$ $\therefore 4 \text{ girls/dogters}$	✓ 26,46 ✓ answer (2)
2.3	 <p>12 14 16 18 20 22 24 26 28 30 36</p>	✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / Meisies	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) None of the boys / Nie een van die seuns nie 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. 5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.	✓ answer ✓ reason/rede (2) [13]

QUESTION/VRAAG 3



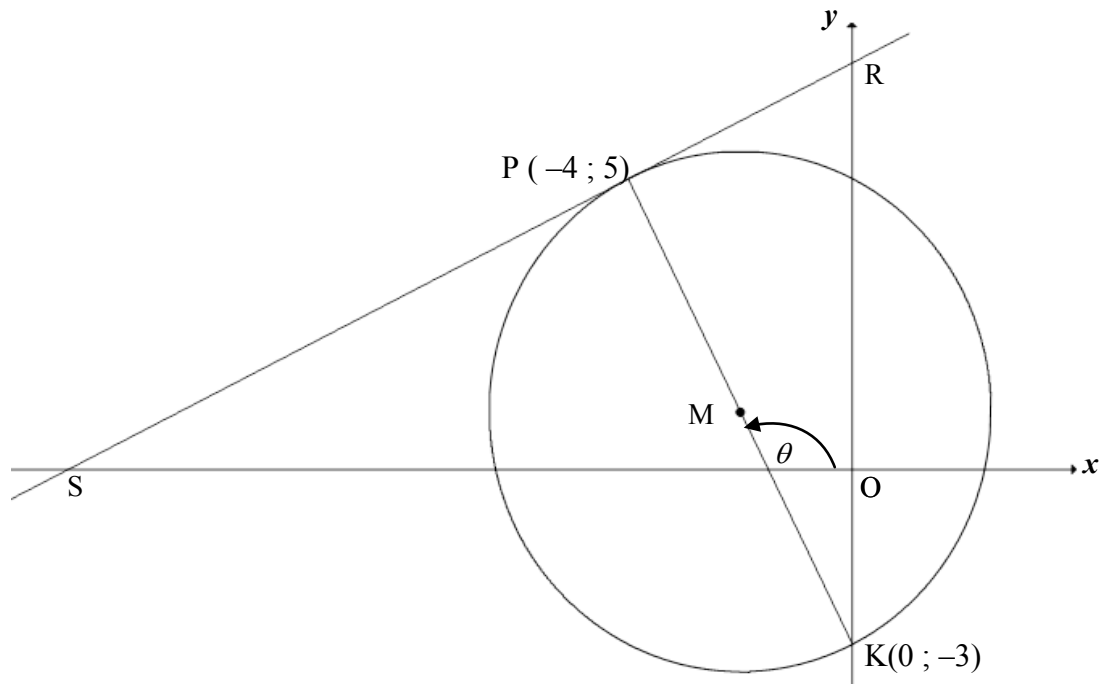
<p>3.1.1</p>	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3\frac{1}{2} - (-4)}{3 - 8}$ $= -\frac{3}{2}$ <p>$y = mx + c$ $y - y_1 = m(x - x_1)$</p> $y = -\frac{3}{2}x + c$ $-4 = -\frac{3}{2}(8) + c \quad \text{OR/OF} \quad (y - (-4)) = -\frac{3}{2}(x - 8)$ $c = 8$ $y = -\frac{3}{2}x + 8$ <p>OR/OF</p> $y + 4 = -\frac{3}{2}x + 12$ $y = -\frac{3}{2}x + 8$	<ul style="list-style-type: none"> ✓ substitution of (8 ; -4) & $(3; 3\frac{1}{2})$ ✓ gradient ✓ substitution of m and (8 ; -4) ✓ equation of AC <p style="text-align: right;">(4)</p>
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	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - \left(3\frac{1}{2}\right)}{8 - 3}$ $= -\frac{3}{2}$ $y = mx + c$ $3\frac{1}{2} = -\frac{3}{2}(3) + c$ $c = 8$ $y = -\frac{3}{2}x + 8$ $y - y_1 = m(x - x_1)$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}(x - 3)$ <p style="text-align: center;">OR/OF</p> $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}x + \frac{9}{2}$ $y = -\frac{3}{2}x + 8$	<ul style="list-style-type: none"> ✓ substitution of $(8 ; -4)$ & $\left(3 ; 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and $\left(3 ; 3\frac{1}{2}\right)$ ✓ equation of AC <p style="text-align: right;">(4)</p>
<p>3.1.2</p>	<p>AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$</p> $15x + 10y = 80$ $\underline{7x - 10y = 8}$ $22x = 88$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ <p>∴ G(4 ; 2)</p> <p>OR/OF</p> <p>BG: $7x - 10y = 8$ ∴ $y = \frac{7}{10}x - \frac{8}{10}$</p> $\therefore \frac{7}{10}x - \frac{8}{10} = -\frac{3}{2}x + 8 \quad [\text{CA from 3.1.1}]$ $\frac{11}{5}x = \frac{44}{5}$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ <p>∴ G(4 ; 2)</p>	<ul style="list-style-type: none"> ✓ method /metode: solving simultaneously / los gelyktydig op ✓ x coordinate ($x > 0$) ✓ y coordinate <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ method: equating metode: stel vgl's gelyk ✓ x coordinate ($x > 0$) ✓ y coordinate <p style="text-align: right;">(3)</p>
<p>3.2</p>	$\frac{x_A + 4}{2} = 3 \quad \text{and} \quad \frac{y_A + 2}{2} = 3\frac{1}{2}$ <p>∴ A(2 ; 5)</p> <p>OR/OF by translation/deur translasie:</p> $x_A = 3 - (4 - 3) = 2$ $y_A = 3\frac{1}{2} + (3\frac{1}{2} - 2) = 5$ <p>∴ A(2 ; 5)</p>	<ul style="list-style-type: none"> ✓ equation ito x ✓ equation ito y <p style="text-align: right;">(2)</p> <ul style="list-style-type: none"> ✓ equation ito x ✓ equation ito y <p style="text-align: right;">(2)</p>

<p>3.3</p>	<p>The coordinates of the midpt of AB / <i>Die koordinaat van midpt van AB is:</i></p> $\left(\frac{2+(-6)}{2}; \frac{5+(-5)}{2}\right) = (-2; 0)$ <p>But the y-coordinate of E is 0 ∴ E(-2 ; 0) is the midpoint of AB ∴ EF BG [midpoint theorem/<i>middelpuntst</i> OR/OF line divides 2 sides of Δ in prop/lyn <i>verdeel 2 sye van Δ in dies verh</i>]</p> <p>OR/OF The coordinates of the midpt of AB / <i>Die koordinaat van midpt van AB is:</i></p> $\left(\frac{2+(-6)}{2}; \frac{5+(-5)}{2}\right) = (-2; 0)$ $AE = \sqrt{(-2-2)^2 + (0-5)^2} = \sqrt{41}$ $EB = \sqrt{(-2-(-6))^2 + (0-(-5))^2} = \sqrt{41}$ <p>∴ In ΔABG: AE = EB and AF = FG ∴ EF BG [midpoint theorem/<i>middelpuntst</i>]</p> <p>OR/OF Equation of AB:</p> $y - (-5) = \left(\frac{5-(-5)}{2-(-6)}\right)(x - (-6))$ $y + 5 = \frac{10}{8}x + \frac{15}{2} \quad \therefore y = \frac{5}{4}x + \frac{5}{2}$ <p>x-intercept of AB:</p> $0 = \frac{5}{4}x + \frac{5}{2} \quad \therefore x = -2$ <p>∴ E(-2 ; 0)</p> $m_{EF} = \frac{3\frac{1}{2} - 0}{3 - (-2)} = \frac{7}{10}$ $m_{EF} = m_{BG} = \frac{7}{10}$ <p>∴ EF BG</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>BG: $7x - 10y = 8$ $\therefore y = \frac{7}{10}x - \frac{8}{10}$ $\therefore m_{BG} = \frac{7}{10}$</p> </div>	<ul style="list-style-type: none"> ✓ subst A & B into midpt formula ✓ y coordinate = 0 ✓ E = midpt ✓ Reason (4) <ul style="list-style-type: none"> ✓ subst A & B into midpt formula ✓ lengths of AE & EB ✓ AE = EB or E = midpt ✓ Reason (4) <ul style="list-style-type: none"> ✓ equation of AB ✓ coordinates of E ✓ gradient of EF ✓ gradient EF = gradient BG (4)
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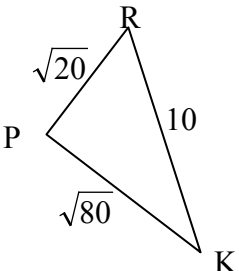
3.4	<p>Midpoint of AC = $\left(5; \frac{1}{2}\right)$</p> $\frac{x_D + (-6)}{2} = 5 \quad \text{and} \quad \frac{y_D + (-5)}{2} = \frac{1}{2}$ <p>$\therefore D(16; 6)$</p> <p>OR/OF by translation/<i>dmv translasië</i>: D(16; 6)</p> <p>OR/OF</p> $m_{BC} = \frac{-5 - (-4)}{-6 - 8} = \frac{1}{14} \quad \text{and} \quad m_{AB} = \frac{5 - (-5)}{2 - (-6)} = \frac{5}{4}$ <p>AD: $y - 5 = \frac{1}{14}(x - 2) \Rightarrow y = \frac{1}{14}x + \frac{34}{7}$</p> <p>CD: $y + 4 = \frac{5}{4}(x - 8) \Rightarrow y = \frac{5}{4}x - 14$</p> $\frac{5}{4}x - 14 = \frac{1}{14}x + \frac{34}{7}$ <p>$\therefore x = 16$ $y = 6$</p>	<p>$\checkmark\checkmark \left(5; \frac{1}{2}\right)$</p> <p>$\checkmark$ x value \checkmark y value (4)</p> <p>\checkmark method finding x \checkmark method finding y \checkmark x value \checkmark y value (4)</p> <p>$\checkmark\checkmark$ equating \checkmark x value \checkmark y value (4)</p> <p>[17]</p>
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QUESTION/VRAAG 4



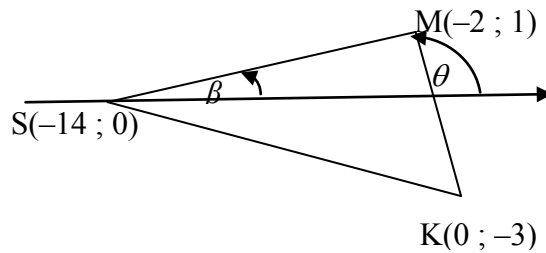
<p>4.1.1</p>	$m_{PK} = \frac{5 - (-3)}{-4 - 0}$ $= -2$ <p>PK \perp SR [radius \perp tangent/raaklyn]</p> $\therefore m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	<ul style="list-style-type: none"> ✓ substitution P & K into gradient formula ✓ gradient of PK ✓ PK \perp SR OR r \perp tangent ✓ answer <p style="text-align: right;">(4)</p>
<p>4.1.2</p>	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c \quad \mathbf{OR/OR} \quad (y - 5) = \frac{1}{2}(x - (-4))$ $c = 7 \quad (y - 5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7 \quad y = \frac{1}{2}x + 7$	<ul style="list-style-type: none"> ✓ substitution of m and P ✓ equation <p style="text-align: right;">(2)</p>

<p>4.1.3</p>	$M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right)$ $\therefore M(-2; 1)$ $r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $r^2 = (-2 + 4)^2 + (1 - 5)^2$ $\therefore r^2 = 20$ $\therefore (x + 2)^2 + (y - 1)^2 = 20 \text{ or } (\sqrt{20})^2$ <p>OR/OF</p> $M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right) \therefore M(-2; 1)$ $(x + 2)^2 + (y - 1)^2 = r^2$ $(-4 + 2)^2 + (5 - 1)^2 = r^2$ $\therefore r^2 = 20$ $\therefore (x + 2)^2 + (y - 1)^2 = 20 \text{ or } (\sqrt{20})^2$ <p>OR/OF</p> $M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right) \therefore M(-2; 1)$ $PK = \sqrt{(-4 - 0)^2 + (5 - (-3))^2} = \sqrt{80}$ $r = \frac{\sqrt{80}}{2} = \sqrt{20}$ $\therefore (x + 2)^2 + (y - 1)^2 = 20 \text{ or } (\sqrt{20})^2$	<p>✓ x value of M ✓ y value of M</p> <p>✓ $r^2 = 20$</p> <p>✓ equation (4)</p> <p>✓✓ M (- 2 ; 1)</p> <p>$r^2 = 20$</p> <p>✓ equation (4)</p> <p>✓✓ M (- 2 ; 1)</p> <p>$r^2 = 20$</p> <p>✓ equation (4)</p>
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<p>4.1.4</p>	<p> $\tan \theta = m_{PK} = -2$ $\therefore \theta = 180^\circ - 63,43^\circ$ $= 116,57^\circ$ $\hat{P}KR = 116,57^\circ - 90^\circ$ [ext \angle of ΔMOK] $= 26,57^\circ$ OR/OF </p>  <p> <u>In ΔRPK:</u> $PK = \sqrt{(0 - (-4))^2 + (-3 - 5)^2} = \sqrt{80}$ $PR = \sqrt{(-4 - 0)^2 + (5 - 7)^2} = \sqrt{20}$ $RK = 10$ $\cos \hat{P}KR = \frac{PK^2 + KR^2 - PR^2}{2 \cdot PK \cdot KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $\hat{P}KR = 26,57^\circ$ OR/OF </p> <p> $\sin \hat{P}KR = \frac{\sqrt{20}}{10}$ OR/OF $\cos \hat{P}KR = \frac{\sqrt{80}}{10}$ $\hat{P}KR = 26,57^\circ$ $\hat{P}KR = 26,57^\circ$ OR/OF </p> <p> $\tan \hat{P}KR = \frac{\sqrt{20}}{\sqrt{80}}$ $\hat{P}KR = 26,57^\circ$ </p>	<p> $\checkmark \tan \theta = -2$ \checkmark size of θ \checkmark answer (3) </p> <p> \checkmark lengths of PK, PR & RK \checkmark correct values into cos rule \checkmark answer (3) </p> <p> \checkmark lengths of sides \checkmark ratio \checkmark answer (3) </p> <p> \checkmark lengths of sides \checkmark ratio \checkmark answer (3) </p>
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4.1.5	<p>RS tangent at K(0 ; -3)</p> $\therefore m_{PS} = m_{\text{tang}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ <p>OR/OF</p> $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{\text{tang}} = -1 \quad [\text{radius } \perp \text{ tangent/raaklyn}]$ $\therefore m_{\text{tang}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	<p>✓ gradient</p> <p>✓ equation (2)</p> <p>✓ gradient</p> <p>✓ equation (2)</p>
4.2	<p>$t \in (-3 ; 7)$</p> <p>OR/OF</p> $-3 < t < 7$	<p>✓ -3 (A)</p> <p>✓ 7 (CA from 4.1.2)</p> <p>✓ correct inequality (3)</p> <p>✓ -3 (A)</p> <p>✓ 7 (CA from 4.1.2)</p> <p>✓ correct inequality (3)</p>
4.3	<p>RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14 ; 0)$</p> $SP = \sqrt{(-14 - (-4))^2 + (0 - 5)^2} = \sqrt{100 + 25} = \sqrt{125}$ $\text{Area } \triangle SMK = \frac{1}{2} \cdot MK \cdot SP$ $= \frac{1}{2} (\sqrt{20})(\sqrt{125})$ $= 25 \text{ square units}$	<p>✓ coordinates of S</p> <p>✓ length of SP</p> <p>✓ correct base & height into Area rule</p> <p>✓ correct substitution</p> <p>✓ answer (5)</p>

OR/OF



Let β = inclination of SM/ *inklinasie van SM*

RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$

$SM = \sqrt{(-14 - (-2))^2 + (0 - 1)^2} = \sqrt{145}$

$\tan \beta = \frac{1 - 0}{-2 - (-14)} = \frac{1}{12} \quad \therefore \beta = 4,76^\circ$

$\therefore \hat{SMK} = 116,57^\circ - 4,76^\circ \quad [\text{ext } \angle \text{ of } \Delta]$
 $= 111,81^\circ$

Area $\Delta SMK = \frac{1}{2}(SM)(MK) \cdot \sin \hat{SMK}$
 $= \frac{1}{2}(\sqrt{145})(\sqrt{20}) \cdot \sin 111,81^\circ$
 $= 24,9985 = 25 \text{ square units}$

✓ coordinates of S

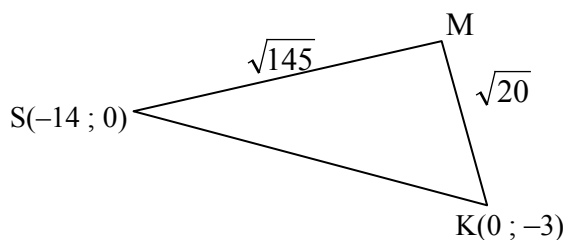
✓ length of SM

✓ size of/grootte v \hat{SMK}

✓ correct substitution into area rule
 ✓ answer

(5)

OR/OF



RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$

$SK = \sqrt{(-14 - 0)^2 + (0 + 3)^2} = \sqrt{205}$

$\cos \hat{SMK} = \frac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\frac{2\sqrt{29}}{29}$

$\hat{SMK} = 111,80^\circ$

Area $\Delta SMK = \frac{1}{2}(SM)(MK) \cdot \sin \hat{SMK}$
 $= \frac{1}{2}(\sqrt{145})(\sqrt{20}) \cdot \sin 111,81^\circ$
 $= 24,9985 = 25 \text{ square units}$

✓ coordinates of S

✓ length of SK

✓ size of/grootte v \hat{SMK}

✓ correct substitution into area rule
 ✓ answer

(5)

	<p>OR/OF</p> <p>Produce KS to T</p> <p>RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$</p> <p>$SK = \sqrt{(-14 - 0)^2 + (0 + 3)^2} = \sqrt{205}$</p> <p>$SM = \sqrt{(-14 - (-2))^2 + (0 - 1)^2} = \sqrt{145}$</p> <p>$m_{SK} = -\frac{3}{14} \Rightarrow T\hat{S}O = 167,91^\circ$</p> <p>$m_{SM} = \frac{1}{12} \Rightarrow M\hat{S}O = 4,76^\circ$</p> <p>$M\hat{S}K = 180^\circ - 167,91^\circ + 4,76^\circ = 16,85^\circ$</p> <p>Area $\Delta SMK = \frac{1}{2}(SM)(SK) \cdot \sin M\hat{S}K$</p> <p>$= \frac{1}{2}(\sqrt{145})(\sqrt{205}) \cdot \sin 16,85^\circ$</p> <p>$= 24,9985 = 25 \text{ square units}$</p>	<p>✓ coordinates of S</p> <p>✓ length of SK & SM</p> <p>✓ size of /grootte v $M\hat{S}K$</p> <p>✓ correct substitution into area rule</p> <p>✓ answer</p> <p>(5)</p>
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QUESTION/VRAAG 5

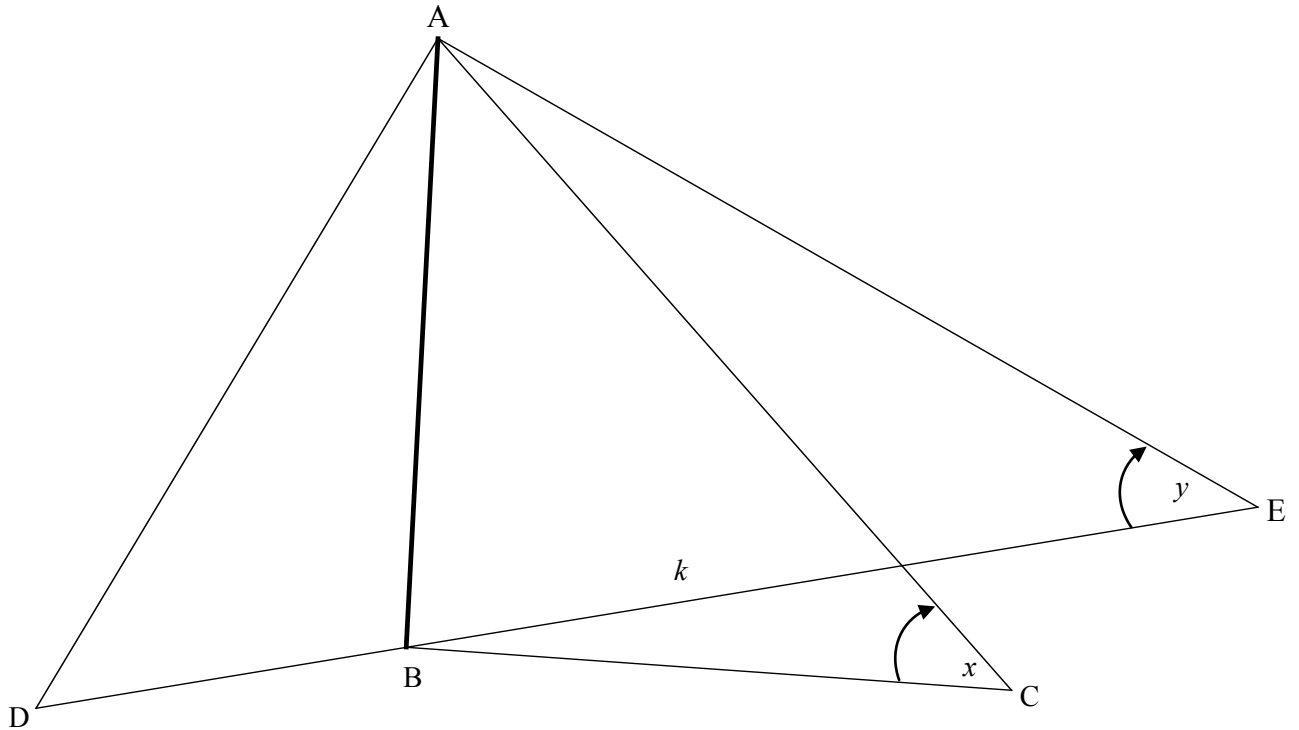
5.1	$\frac{\sin(A - 360^\circ) \cdot \cos(90^\circ + A)}{\cos(90^\circ - A) \cdot \tan(-A)}$ $= \frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $= \frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$ $= \cos A$	<ul style="list-style-type: none"> ✓ sin A ✓ -sin A ✓ sin A ✓ -tan A ✓ $\tan A = \frac{\sin A}{\cos A}$ ✓ answer 	(6)
5.2.1	$t^2 = (\sqrt{34})^2 - (3)^2$ $\therefore t = -5$	<ul style="list-style-type: none"> ✓ substitution ✓ answer 	(2)
5.2.2	$\tan \beta = \frac{-5}{3}$	<ul style="list-style-type: none"> ✓ correct ratio 	(1)
5.2.3	$\cos 2\beta = 2 \cos^2 \beta - 1$ $= 2 \left(\frac{3}{\sqrt{34}} \right)^2 - 1$ $= 2 \left(\frac{9}{34} \right) - 1$ $= -\frac{16}{34} \text{ OR } -\frac{8}{17}$ <p>OR/OF</p> $\cos 2\beta = 1 - 2 \sin^2 \beta$ $= 1 - 2 \left(-\frac{5}{\sqrt{34}} \right)^2$ $= 1 - 2 \left(\frac{25}{34} \right)$ $= -\frac{16}{34} \text{ OR } -\frac{8}{17}$ <p>OR/OF</p> $\cos 2\beta = \cos^2 \beta - \sin^2 \beta$ $= \left(\frac{3}{\sqrt{34}} \right)^2 - \left(-\frac{5}{\sqrt{34}} \right)^2$ $= \frac{9}{34} - \frac{25}{34}$ $= -\frac{16}{34} \text{ OR } -\frac{8}{17}$	<ul style="list-style-type: none"> ✓ compound formula ✓ substitution ✓ simplification ✓ answer 	(4)
		<ul style="list-style-type: none"> ✓ compound formula ✓ substitution ✓ simplification ✓ answer 	(4)
		<ul style="list-style-type: none"> ✓ compound formula ✓ substitution ✓ simplification ✓ answer 	(4)

<p>5.3.1</p>	<p>LHS = $\sin(A + B) - \sin(A - B)$ $= \sin A \cdot \cos B + \cos A \cdot \sin B - (\sin A \cdot \cos B - \cos A \cdot \sin B)$ $= \sin A \cdot \cos B + \cos A \cdot \sin B - \sin A \cdot \cos B + \cos A \cdot \sin B$ $= 2\cos A \cdot \sin B$ $= \text{RHS}$</p>	<p>✓ compound formula ✓ compound formula (2)</p>
<p>5.3.2</p>	<p>$\sin 77^\circ - \sin 43^\circ = \sin(60^\circ + 17^\circ) - \sin(60^\circ - 17^\circ)$ $= 2\cos 60^\circ \cdot \sin 17^\circ$ $= 2 \times \frac{1}{2} \times \sin 17^\circ$ $= \sin 17^\circ$</p> <p>OR/OF</p> <p>$\sin 77^\circ - \sin 43^\circ = \sin(60^\circ + 17^\circ) - \sin(60^\circ - 17^\circ)$ $= (\sin 60^\circ \cos 17^\circ + \cos 60^\circ \sin 17^\circ) -$ $(\sin 60^\circ \cos 17^\circ - \cos 60^\circ \sin 17^\circ)$ $= \frac{\sqrt{3}}{2} \cos 17^\circ + \frac{1}{2} \sin 17^\circ - \frac{\sqrt{3}}{2} \cos 17^\circ + \frac{1}{2} \sin 17^\circ$ $= \sin 17^\circ$</p>	<p>✓ $60^\circ + 17^\circ$ ✓ $60^\circ - 17^\circ$ ✓ simplify ✓ $\frac{1}{2}$ (4)</p> <p>✓ $60^\circ + 17^\circ$ ✓ $60^\circ - 17^\circ$ ✓ expansion ✓ $\frac{1}{2}$ (4) [19]</p>

QUESTION/VRAAG 6

<p>6.1</p>		<ul style="list-style-type: none"> ✓ $(-90^\circ ; -3)$ ✓ $(0 ; -1)$ ✓ x – intercepts: -210° & 30° ✓ shape <p style="text-align: right;">(4)</p>
<p>6.2</p>	$\cos 2x = 2 \sin x - 1$ $1 - 2 \sin^2 x = 2 \sin x - 1$ $2 \sin^2 x + 2 \sin x - 2 = 0$ $\sin^2 x + \sin x - 1 = 0$ $\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$ $\sin x = \frac{-1 + \sqrt{5}}{2}, \text{ since } \sin x = \frac{-1 - \sqrt{5}}{2} < -1 \text{ has no solution}$	<ul style="list-style-type: none"> ✓ $\cos 2x = 1 - 2 \sin^2 x$ ✓ standard form ✓ using quadratic formula ✓ substitution into quadratic formula <p style="text-align: right;">(4)</p>
<p>6.3</p>	$\sin x = \frac{-1 + \sqrt{5}}{2} = 0,618\dots$ <p>Reference $\angle = 38,17^\circ$</p> <p>$\therefore x = 38,17^\circ + k \cdot 360^\circ$ or $x = 141,83^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$</p> <p>$\therefore x = 38,17^\circ$ or $-218,17^\circ$</p> <p>$y = 0,24$</p> <p>\therefore Points of intersection/snyppunte: $(38,17^\circ ; 0,24)$ and $(-218,17^\circ ; 0,24)$</p>	<ul style="list-style-type: none"> ✓ $38,17^\circ$ ✓ $141,83^\circ$ ✓ $-218,17^\circ$ ✓ $0,24$ <p style="text-align: right;">(4) [12]</p>

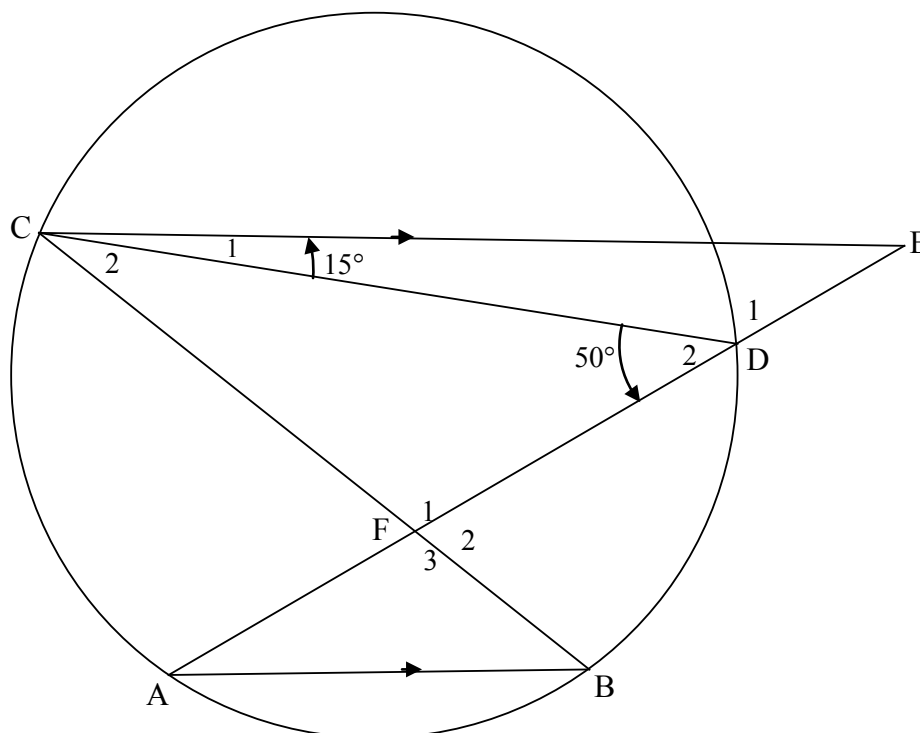
QUESTION/VRAAG 7



7.1	$\hat{A}BC = 90^\circ$	✓ answer (1)
7.2	<p>In $\triangle ABE$:</p> $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ <p>In $\triangle ABC$:</p> $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	<p>✓ correct ratio ✓ value AB</p> <p>✓ correct ratio ✓ AC as subject and substitution</p> <p>(4)</p>

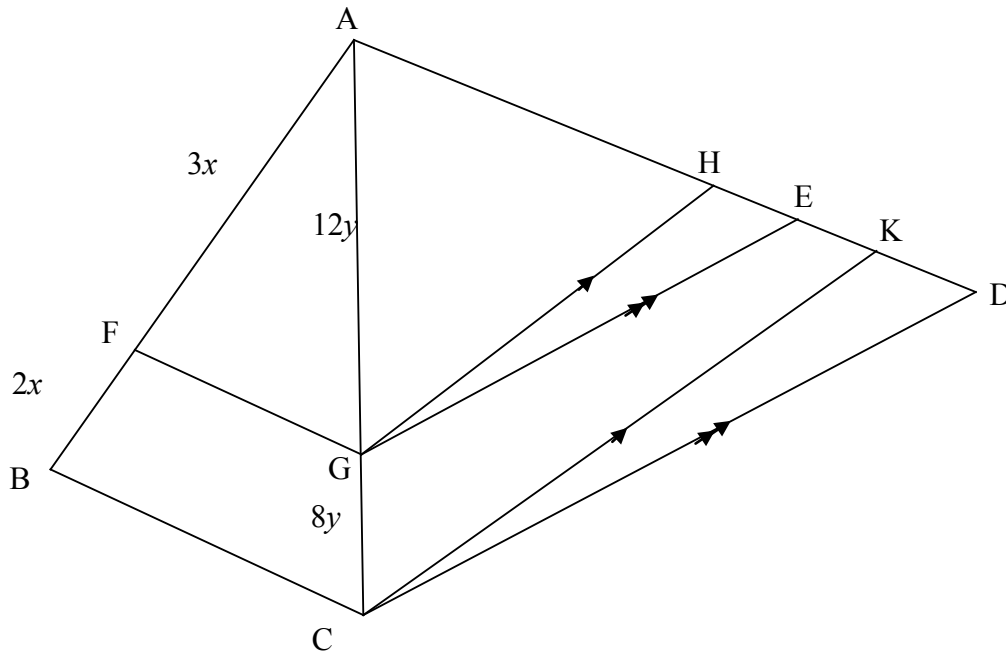
<p>7.3</p>	$\hat{A}DC = \hat{A}CD = \frac{180^\circ - 2x}{2} = 90^\circ - x$ $\frac{DC}{\sin 2x} = \frac{AC}{\sin(90^\circ - x)}$ $\frac{DC}{2 \sin x \cos x} = \frac{AC}{\cos x}$ $DC = \frac{AC(2 \sin x \cos x)}{\cos x}$ $= \frac{k \tan y}{\sin x} \cdot \frac{2 \sin x \cos x}{\cos x}$ $= 2k \tan y$ <p>OR/OF</p> $DC^2 = AD^2 + AC^2 - 2AD \cdot AC \cos 2x$ $= AC^2 + AC^2 - 2AC^2 \cos 2x$ $= 2AC^2(1 - \cos 2x)$ $= 2AC^2(1 - 1 + \sin^2 x)$ $= 4AC^2 \sin^2 x$ $DC = 2AC \cdot \sin x$ $= 2 \left(\frac{k \cdot \tan y}{\sin x} \right) \cdot \sin x$ $= 2k \cdot \tan y$ <p>OR/OF</p> $DC^2 = AD^2 + AC^2 - 2AD \cdot AC \cos 2x$ $= 2 \left(\frac{k \tan y}{\sin x} \right)^2 - 2 \left(\frac{k \tan y}{\sin x} \right)^2 \cos 2x$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x} (1 - 2 \sin^2 x)$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x} + 4k^2 \tan^2 y$ $DC = \sqrt{4k^2 \tan^2 y}$ $= 2k \tan y$	<ul style="list-style-type: none"> ✓ $90^\circ - x$ ✓ subst into sine rule ✓ $2 \sin x \cos x$ ✓ $\cos x$ ✓ substitution (5) ✓ substitution into cos rule ✓ factorisation ✓ $1 - 2 \sin^2 x$ ✓ DC ito AC and $\sin x$ ✓ substitution (5) ✓ correct cos rule ✓ substitution ✓ $1 - 2 \sin^2 x$ ✓ squaring and multiplication ✓ $\sqrt{4k^2 \tan^2 y}$ (5) <p style="text-align: right;">[10]</p>
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QUESTION/VRAAG 8



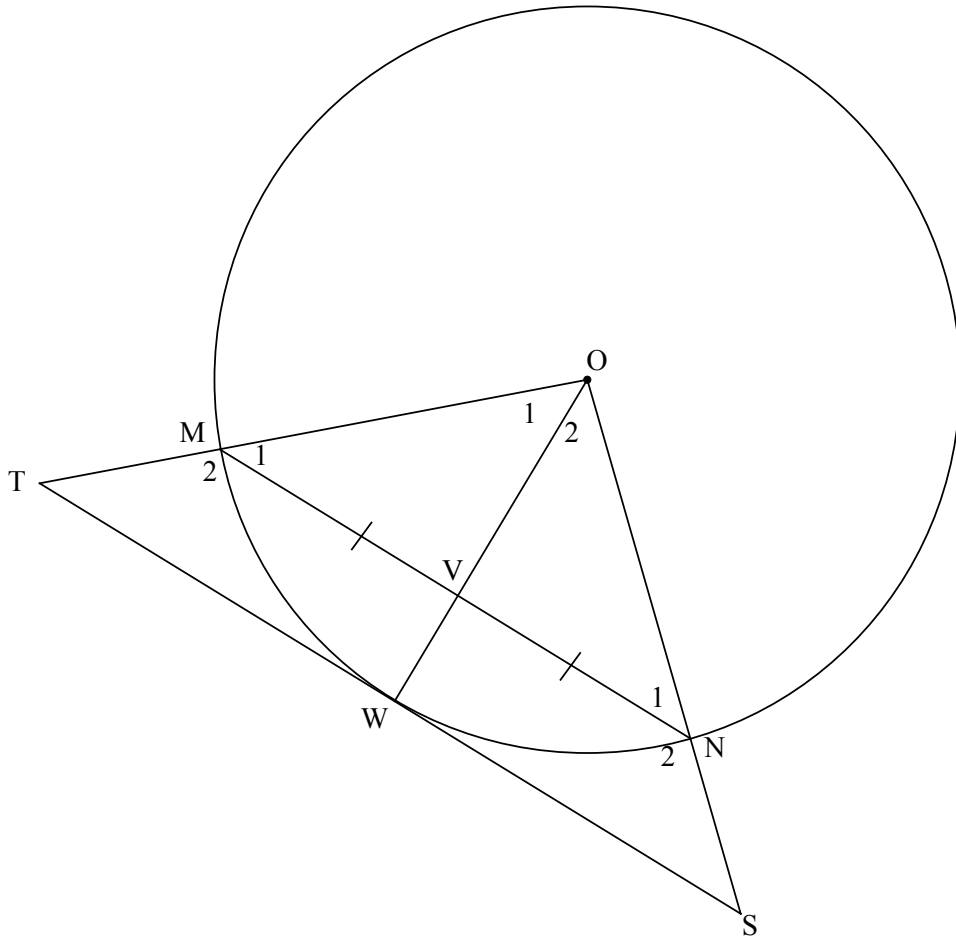
<p>8.1.1</p>	<p>$\hat{E} = 50^\circ - 15^\circ = 35^\circ$ [ext \angle of Δ/buite \angle van Δ] $\hat{A} = 35^\circ$ [alt \angles / verwiss \anglee; CE AB]</p> <p>OR/OF $\hat{E} = 180^\circ - (130^\circ + 15^\circ) = 35^\circ$ [str line; \angles of Δ/rt lyn; \anglee van Δ] $\hat{A} = 35^\circ$ [alt \angles / verwiss \anglee; CE AB]</p> <p>OR/OF $\hat{B} = 50^\circ$ [\angles in same segment/\anglee in dieselfde segment] $\hat{C}_2 + 15^\circ = 50^\circ$ [alt \angles / verwiss \anglee; CE AB] $\therefore \hat{C}_2 = 35^\circ$ $\hat{A} = 35^\circ$ [\angles in same segment/\anglee in dieselfde segment]</p>	<p>✓ S ✓ S ✓ R (3)</p> <p>✓ S ✓ S ✓ R (3)</p> <p>✓ S ✓ S ✓ R (3)</p>
<p>8.1.2</p>	<p>$\hat{C}_2 = 35^\circ$ [\angles in same segment/\anglee in dieselfde segment]</p>	<p>✓ S ✓ R (2)</p>
<p>8.2</p>	<p>$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] \therefore CF is a tangent to the circle [converse tan chord theorem] \therefore CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]</p>	<p>✓ S ✓ R (2) [7]</p>

QUESTION/VRAAG 9



<p>9.1.1</p>	$\frac{AF}{BF} = \frac{3x}{2x} = \frac{3}{2} \quad \& \quad \frac{AG}{CG} = \frac{12y}{8y} = \frac{3}{2}$ $\therefore \frac{AF}{BF} = \frac{AG}{CG}$ <p>$\therefore FG \parallel BC$ [conv prop th/omg eweredigh st. OR line divides 2 sides of Δ in prop/lyn verdeel 2 sye v Δ in dies verh]</p>	<p>$\checkmark \frac{AF}{BF} = \frac{AG}{CG}$</p> <p>$\checkmark R$</p> <p style="text-align: right;">(2)</p>
<p>9.1.2</p>	$\frac{AG}{GC} = \frac{AH}{HK} \quad \text{[prop theorem/eweredigh st; } \underline{GH \parallel CK} \text{ OR line } \parallel \text{ to 1 side of } \Delta \text{/lyn } \parallel \text{ 1 sy van } \Delta]$ $\frac{AG}{GC} = \frac{AE}{ED} \quad \text{[prop theorem/eweredigh st; } \underline{GE \parallel CD}]$ $\therefore \frac{AH}{HK} = \frac{AE}{ED}$	<p>$\checkmark S \checkmark R$</p> <p>$\checkmark S$</p> <p style="text-align: right;">(3)</p>
<p>9.2</p>	$\frac{AE}{ED} = \frac{3}{2} \quad \text{and} \quad \frac{AH}{HK} = \frac{3}{2}$ $\frac{AE}{12} = \frac{3}{2} \quad \text{and} \quad \frac{15}{HK} = \frac{3}{2}$ <p>$\therefore AE = 18$ and $HK = 10$</p> <p>$\therefore HE = AE - AH$ $= 18 - 15$ $= 3$</p> <p>$\therefore EK = HK - HE$ $= 10 - 3$ $= 7$</p> <p style="text-align: center;">OR/OF</p> $AD = 30$ $KD = AD - AH - HK$ $= 30 - 15 - 10$ $= 5$ $EK = ED - KD$ $= 12 - 5$ $= 7$	<p>\checkmark use of ratios</p> <p>$\checkmark AE = 18$</p> <p>$\checkmark HK = 10$</p> <p>$\checkmark HE = 3$ or $KD = 5$</p> <p>$\checkmark EK = 7$</p> <p style="text-align: right;">(5) [10]</p>

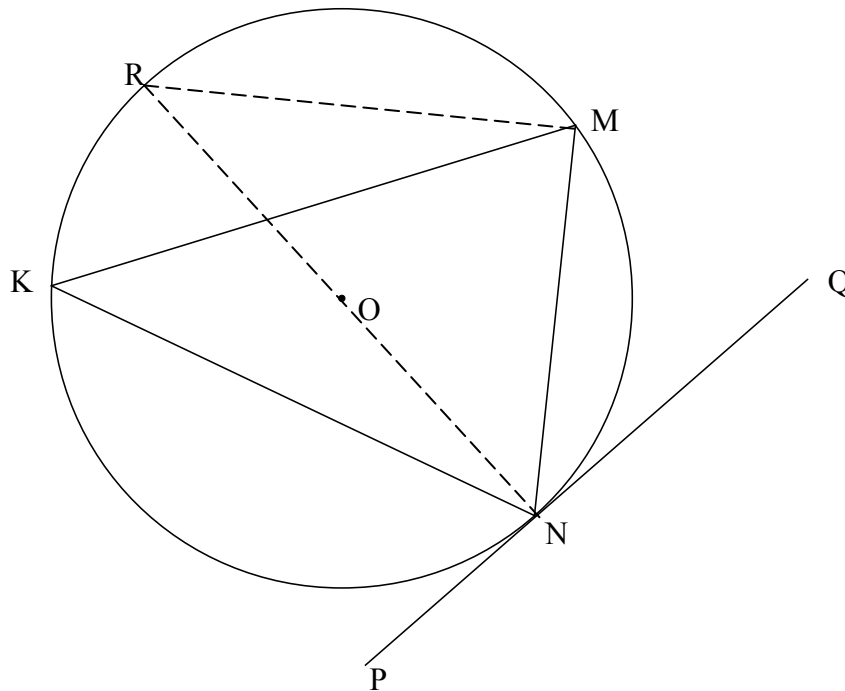
QUESTION/VRAAG 10



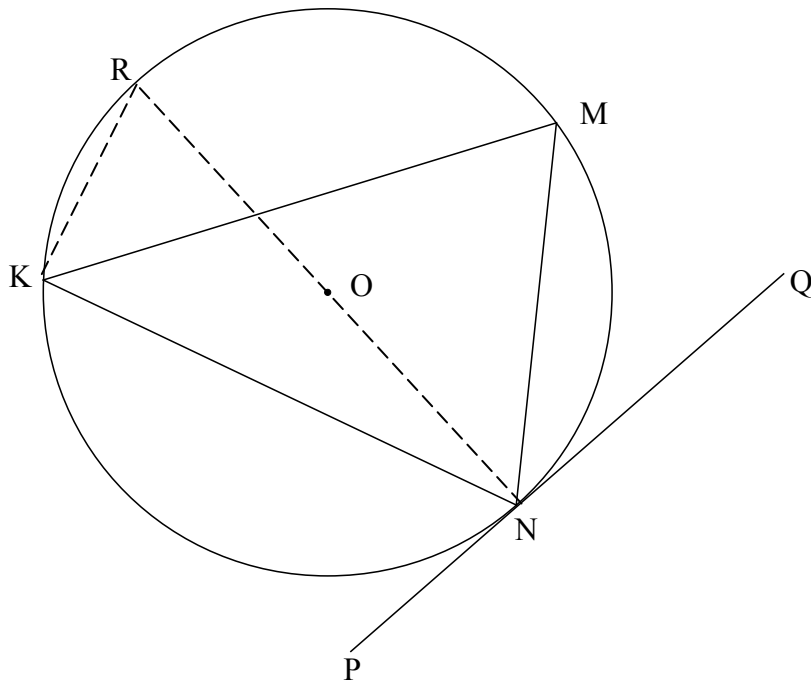
10.1	Line from centre to midpoint of chord/ <i>lyn vanaf midpt na midpt van koord</i>	✓ R	(1)
10.2.1	$\widehat{O\hat{W}T} = \widehat{O\hat{W}S} = 90^\circ$ [radius \perp tangent/ <i>raaklyn</i>] $\therefore MN \parallel TS$ [corresp \angle s = /ooreenkomstige \angle e = OR co-int \angle s 180°/ko-binne \angle e 180° OR alternate \angle s/ <i>verwiss \anglee</i>]	✓ R ✓ R	(2)
10.2.2	$\hat{M}_1 = \hat{N}_1$ [\angle s opp = sides/ \angle e teenoor = sye] $\hat{M}_1 = \hat{T}$ [corresp \angle s/ooreenk \angle e; $MN \parallel TS$] $\therefore \hat{N}_1 = \hat{T}$ \therefore TMNS is a cyclic quadrilateral [conv: ext \angle cyclic quad] TMNS is 'n koordevierhoek [<i>omgek: buite \angle kdvh</i>] OR/OF $\hat{M}_1 = \hat{N}_1$ [\angle s opp = sides/ \angle e teenoor = sye] $\hat{N}_1 = \hat{S}$ [corresp \angle s/ooreenk \angle e; $MN \parallel TS$] $\therefore \hat{S} = \hat{M}_1$ \therefore TMNS is a cyclic quadrilateral [conv: ext \angle cyclic quad] TMNS is 'n koordevierhoek [<i>omgek: buite \angle kdvh</i>]	✓ S ✓ S ✓ S ✓ R ✓ S ✓ S ✓ S ✓ R	(4) (4)

<p>10.2.3</p>	<p>In $\triangle OVN$ and $\triangle OWS$ $\hat{O}_2 = \hat{O}_2$ [common/<i>gemeenskaplik</i>] $O\hat{V}N = O\hat{W}S = 90^\circ$ [from 10.1] $O\hat{N}V = O\hat{S}W$ [sum $\angle s \triangle / som \angle e \triangle$] $\therefore \triangle OVN \parallel \triangle OWS$ [\angle, \angle, \angle] $\therefore \frac{VN}{WS} = \frac{ON}{OS}$ But $VN = \frac{1}{2} MN$ [given] $\therefore \frac{\frac{1}{2} MN}{WS} = \frac{ON}{OS}$ $\therefore OS \cdot MN = 2ON \cdot WS$</p> <p>OR/OF In $\triangle OVM$ and $\triangle OWS$ $O\hat{V}M = O\hat{W}S = 90^\circ$ [from 10.1] $O\hat{M}V = O\hat{S}W$ [sum $\angle s \triangle / som \angle e \triangle$] $\therefore \triangle OVM \parallel \triangle OWS$ [\angle, \angle, \angle] $\therefore \frac{OM}{OS} = \frac{VM}{WS}$ But $VN = \frac{1}{2} MN$ [given] $\therefore \frac{\frac{1}{2} MN}{WS} = \frac{OM}{OS}$ $\therefore OS \cdot MN = 2ON \cdot WS$ [VM = VN]</p> <p>OR/OF If any other 2 $\triangle s$ are used, first need to prove that TW = WS by proving $\triangle OWT \equiv \triangle OWS$ In $\triangle OVM$ and $\triangle OWT$ $\hat{O}_1 = \hat{O}_1$ [common/<i>gemeenskaplik</i>] $O\hat{V}M = O\hat{W}T = 90^\circ$ [from 10.1] $O\hat{M}V = O\hat{T}W$ [sum $\angle s \triangle / som \angle e \triangle$] $\therefore \triangle OVM \parallel \triangle OWT$ [\angle, \angle, \angle] $\therefore \frac{VM}{WT} = \frac{OM}{OT}$ But $VN = VM = \frac{1}{2} MN$ [given] and $WT = WS$ and $OT = OS$ [$\triangle OWT \equiv \triangle OWS$] $\therefore \frac{\frac{1}{2} MN}{WS} = \frac{ON}{OS}$ $\therefore OS \cdot MN = 2ON \cdot WS$</p>	<p>✓ S; S; S OR S; S; R</p> <p>✓ $\triangle OVN \parallel \triangle OWS$ ✓ $\frac{VN}{WS} = \frac{ON}{OS}$ ✓ $VN = \frac{1}{2} MN$</p> <p>✓ substitution</p> <p>(5)</p> <p>✓ S; S; S OR S; S; R</p> <p>✓ $\triangle OVM \parallel \triangle OWS$ ✓ $\frac{OM}{OS} = \frac{VM}{WS}$ ✓ $VN = \frac{1}{2} MN$</p> <p>✓ substitution</p> <p>(5)</p> <p>✓ ✓ similarity ✓ ✓ congruency</p> <p>✓ $VN = VM = \frac{1}{2} MN$</p> <p>(5)</p> <p>[12]</p>
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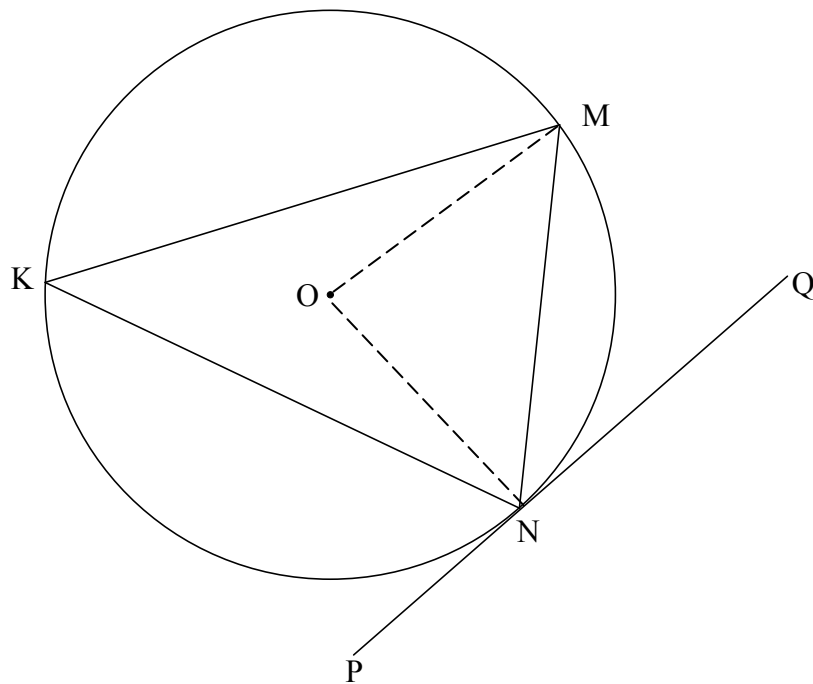
QUESTION/VRAAG 11



<p>11.1</p>	<p>Construction: Draw diameter NR and draw RM <i>Konstruksie: Trek middellyn NR en verbind RM</i> $\widehat{ONM} + \widehat{MNQ} = 90^\circ$ [radius \perp tangent/raaklyn] $\widehat{NMR} = 90^\circ$ [\angle in semi circle/semi-sirkel] $\therefore \widehat{MRN} = 180^\circ - (90^\circ + 90^\circ - \widehat{MNQ})$ [sum \angles Δ] $= \widehat{MNQ}$ but $\widehat{MRN} = \widehat{MKN}$ [\angles same segment/\anglee dieselfde segment] $\therefore \widehat{MNQ} = \widehat{MKN}$ OR/OF</p>	<p>✓ construction ✓ S / R ✓ S / R ✓ S ✓ S / R (5)</p>
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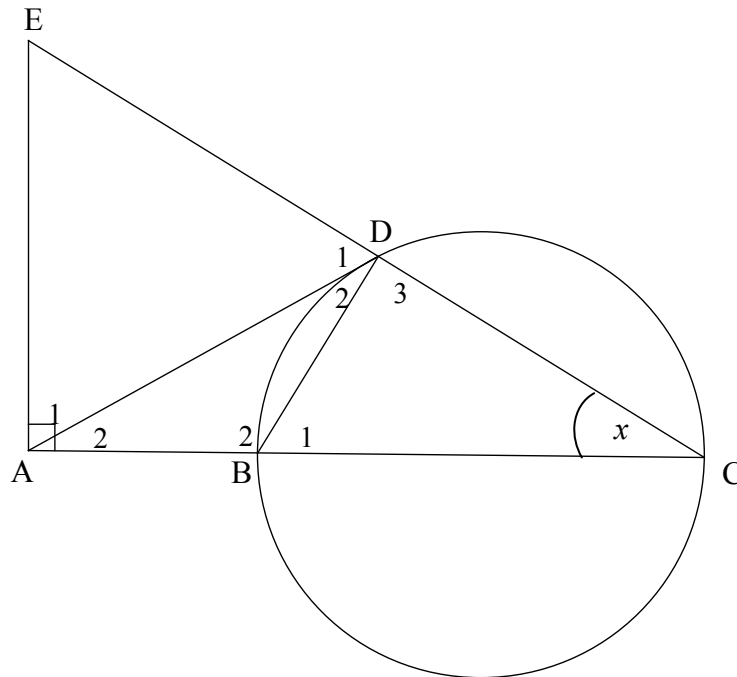


<p>11.1</p>	<p>Construction: Draw diameter NR and draw RK <i>Konstruksie: Trek middellyn NR en verbind RK</i> $M\hat{N}Q + R\hat{N}M = 90^\circ$ [radius \perp tangent/raaklyn] $N\hat{K}R = 90^\circ$ [\angle in semicircle/semi-sirkel] $\therefore M\hat{K}N = 90^\circ - R\hat{K}M$ $= 90^\circ - R\hat{N}M$ [\angles same segment/\anglee dieselfde segment] $\therefore M\hat{N}Q = \hat{K}$</p>	<p>✓ construction ✓ S / R ✓ S / R ✓ S ✓ S / R (5)</p>
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<p>11.1</p>	<p>Construction: Draw radii ON and OM <i>Konstruksie: Trek radiusse ON en OM</i> $\widehat{M\hat{O}N} = 2\hat{K}$ [\angle at centre = $2\angle$ at circumf/midpts $\angle = 2$ omtreks \angle] $\widehat{O\hat{N}M} + \widehat{O\hat{M}N} = 180^\circ - 2\hat{K}$ [\angles of Δ/ \anglee van Δ] $\widehat{O\hat{N}M} = \widehat{O\hat{M}N} = \frac{180^\circ - 2\hat{K}}{2} = 90^\circ - \hat{K}$ [\angles opp = sides/ \anglee teenoor = sye] $\widehat{O\hat{N}Q} = 90^\circ$ [radius \perp tangent/ radius \perp raaklyn] $\therefore \widehat{M\hat{N}Q} = \hat{K}$</p>	<p>✓ construction ✓ S / R ✓ S ✓ S / R ✓ S / R (5)</p>
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11.2



11.2.1(a)	Angle in a semi circle/ <i>Hoek in halfsirkel</i>	✓ R (1)
11.2.1(b)	Exterior \angle of quad = opp interior \angle / <i>Buite \angle van vierh = teenoorst binne \angle</i> OR/OF Opp \angle s of quad supplementary/ <i>Teenoorst \anglee van vierh is supplementêr</i>	✓ R (1)
11.2.1(c)	tangent chord theorem/ <i>raaklyn koord stelling</i>	✓ R (1)
11.2.2(a)	In $\triangle AEC$ $\hat{E} = 180^\circ - (90^\circ + x)$ [sum \angle s \triangle] $= 90^\circ - x$ $\hat{D}_1 = 180^\circ - (90^\circ + x)$ [\angle s on a straight line] $= \hat{E} = 90^\circ - x$ $\therefore AD = AE$ [sides opp = \angle s/ <i>syte teenoor = \anglee</i>]	✓ S ✓ S ✓ R (3)
11.2.2(b)	In $\triangle ADB$ and $\triangle ACD$ $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum \angle \triangle] $\therefore \triangle ADB \parallel \triangle ACD$ OR/OF In $\triangle ADB$ and $\triangle ACD$ $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\therefore \triangle ADB \parallel \triangle ACD$ [\angle , \angle , \angle]	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ R (3)

<p>11.2.3(a)</p>	$\frac{AD}{AC} = \frac{AB}{AD} \quad [\Delta s]$ $AD^2 = AC \cdot AB$ $= 3r \times r$ $= 3r^2$	<p>✓ ratio</p> <p>✓ substitution</p> <p>(2)</p>
<p>11.2.3(b)</p>	<p>$AD = AE = \sqrt{3}r$ [from 11.2.2(a) & 11.2.3(a)]</p> <p>$AB = r$ and $BC = 2r \therefore AC = 3r$</p> <p><u>In ΔACE:</u></p> $\tan \hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ <p>$\therefore \hat{E} = 60^\circ$</p> <p>$\therefore \hat{D}_1 = 60^\circ$ [from 11.2.2(a)]</p> <p>$\therefore \hat{A}_1 = 60^\circ$ [$\angle s$ of $\Delta = 180^\circ$]</p> <p>$\therefore \Delta ADE$ is equilateral/<i>is gelyksydig</i></p> <p>OR/OF</p> $\frac{AD}{AC} = \frac{DB}{CD} \quad [\Delta s]$ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ $\tan x = \frac{1}{\sqrt{3}}$ <p>\therefore In ΔBDC: $x = 30^\circ$</p> <p>$\therefore \hat{E} = 60^\circ$</p> <p>$\therefore \hat{D}_1 = 60^\circ$ [from 11.2.2(a)]</p> <p>$\therefore \hat{A}_1 = 60^\circ$ [$\angle s$ of $\Delta = 180^\circ$]</p> <p>$\therefore \Delta ADE$ is equilateral/<i>is gelyksydig</i></p> <p>OR/OF</p> $\frac{AD}{AC} = \frac{DB}{CD} \quad [\Delta s]$ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD} \quad \therefore BD = \frac{CD}{\sqrt{3}}$ $DC^2 = BC^2 - DB^2$ $= 4r^2 - \frac{CD^2}{3}$ $3DC^2 = 12r^2 - CD^2$ $4CD^2 = 12r^2$ $DC = \sqrt{3}r$	<p>✓ AC ito r</p> <p>✓ trig ratio</p> <p>✓ simplification</p> <p>✓ all 3 $\angle s = 60^\circ$</p> <p>(4)</p> <p>✓ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$</p> <p>✓ $\frac{1}{\sqrt{3}} = \tan x$</p> <p>✓ $x = 30^\circ$</p> <p>✓ all 3 $\angle s = 60^\circ$</p> <p>(4)</p> <p>✓ $BD = \frac{CD}{\sqrt{3}}$</p> <p>✓ $DC = \sqrt{3}r$</p>

	$EC^2 = EA^2 + AC^2$ $= 3r^2 + 9r^2$ $EC = 2\sqrt{3}r$ $\therefore ED = EC - DC$ $= \sqrt{3}r$ $\therefore ED = EA = AD$ $\therefore \triangle ADE \text{ is equilateral/is gelyksydig}$	$\checkmark EC = 2\sqrt{3}r$ $\checkmark ED = EA = AD$ <p style="text-align: right;">(4) [20]</p>
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TOTAL/TOTAAL: 150